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Canceling Quadratic Divergences in a Class of Two-Higgs-Doublet Models

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Abstract

The Newton-Wu conditions for the cancellation of quadratic divergences in a class of two-Higgs-doublet models are analyzed as to how they may be satisfied with a typical extension of the Standard Model of particle interactions.

In the Standard Model (SM) of particle interactions, there is one physical scalar Higgs boson H. Its mass may be constrained by the requirement that the one quadratic divergence of the theory be canceled among its gauge, Yukawa, and quartic scalar couplings [1], i.e.

$$\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{3}{4}m_H^2 = \sum_f N_f m_f^2, \tag{1}$$

where the sum is over all SM fermions with N_f its number of color degrees of freedom, i.e. $N_f = 3$ for quarks and $N_f = 1$ for leptons. The above has been written in terms of masses because all SM couplings are related to masses through the one vacuum expectation value (VEV) of the theory. Given that $m_t = 174$ GeV and all other fermion masses are negligible compared to it, the prediction is then $m_H = 316$ GeV.

In the two-Higgs-doublet extension of the SM, there are 4 quadratic divergences and they are functions again of the gauge, Yukawa, and quartic scalar couplings. They have been calculated previously by Newton and Wu [2]. However, since there are now more possible quartic scalar couplings than masses, and the Yukawa terms are not uniquely determined (because of the different possible choices of which fermion couples to which scalar), the 4 conditions vary according to what specific two-Higgs-doublet extension is assumed.

In this note, the following Higgs potential for two doublets $\Phi_{1,2} = (\phi_{1,2}^+, \phi_{1,2}^0)$ is considered:

$$\begin{aligned} V &= \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \mu_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ &\quad + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \frac{1}{2}\lambda_5^* (\Phi_2^\dagger \Phi_1)^2. \end{aligned} \tag{2}$$

This is the most general form of V under the assumption that a discrete odd-even symmetry (Φ_1 is odd and Φ_2 is even or vice versa) exists which is broken only by soft terms (μ_{12}^2). The utility of such a formulation in model building is widely recognized [3]. The same odd-even symmetry restricts the Yukawa couplings of the SM fermions of each type to be associated with either Φ_1 or Φ_2, but not both. Under these assumptions, the 4 Newton-Wu conditions

reduce to only two:

$$\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{v^2}{2}(3\lambda_1 + 2\lambda_3 + \lambda_4) = \frac{1}{\cos^2\beta}\sum_{f_1} N_{f_1} m_{f_1}^2, \quad (3)$$

$$\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{v^2}{2}(3\lambda_2 + 2\lambda_3 + \lambda_4) = \frac{1}{\sin^2\beta}\sum_{f_2} N_{f_2} m_{f_2}^2, \quad (4)$$

where $\langle\phi_{1,2}^0\rangle = v_{1,2}$, $v^2 = v_1^2 + v_2^2$, and $\tan\beta = v_2/v_1$.

There are 5 physical scalar particles. Assuming λ_5 to be real for simplicity, their masses are given as follows [4]:

$$m_{H^\pm}^2 = -\mu_{12}^2(\tan\beta + \cot\beta) - (\lambda_4 + \lambda_5)v^2, \quad (5)$$

$$m_A^2 = -\mu_{12}^2(\tan\beta + \cot\beta) - 2\lambda_5 v^2, \quad (6)$$

with the other two ($m_{1,2}^2$) being the eigenvalues of the matrix

$$\mathcal{M}^2 = \begin{bmatrix} -\mu_{12}^2\tan\beta + 2\lambda_1 v_1^2 & \mu_{12}^2 + 2(\lambda_3+\lambda_4+\lambda_5)v_1v_2 \\ \mu_{12}^2 + 2(\lambda_3+\lambda_4+\lambda_5)v_1v_2 & -\mu_{12}^2\cot\beta + 2\lambda_2 v_2^2 \end{bmatrix}. \quad (7)$$

In terms of $m_{1,2}^2$ and the mixing angle α which diagonalizes it, the above can be rewritten as

$$\mathcal{M}^2 = \begin{bmatrix} m_1^2\cos^2\alpha + m_2^2\sin^2\alpha & (m_1^2 - m_2^2)\sin\alpha\cos\alpha \\ (m_1^2 - m_2^2)\sin\alpha\cos\alpha & m_2^2\cos^2\alpha + m_1^2\sin^2\alpha \end{bmatrix}. \quad (8)$$

The conditions of Eqs. (3) and (4) can now be formulated as

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} m_1^2 \\ m_2^2 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}, \quad (9)$$

where

$$A_{11} = \frac{3\cos^2\alpha}{\cos^2\beta} + \frac{2\sin\alpha\cos\alpha}{\sin\beta\cos\beta}, \qquad A_{12} = \frac{3\sin^2\alpha}{\cos^2\beta} - \frac{2\sin\alpha\cos\alpha}{\sin\beta\cos\beta}, \quad (10)$$

$$A_{21} = \frac{3\sin^2\alpha}{\sin^2\beta} + \frac{2\sin\alpha\cos\alpha}{\sin\beta\cos\beta}, \qquad A_{22} = \frac{3\cos^2\alpha}{\sin^2\beta} - \frac{2\sin\alpha\cos\alpha}{\sin\beta\cos\beta}, \quad (11)$$

and

$$C_1 = \frac{4}{\cos^2\beta}\sum_{f_1} N_{f_1} m_{f_1}^2 - 6M_W^2 - 3M_Z^2 - 2m_{H^\pm}^2 - m_A^2 - \frac{\mu_{12}^2}{\sin\beta\cos\beta}[1 + 3\tan^2\beta], \quad (12)$$

$$C_2 = \frac{4}{\sin^2\beta}\sum_{f_2} N_{f_2} m_{f_2}^2 - 6M_W^2 - 3M_Z^2 - 2m_{H^\pm}^2 - m_A^2 - \frac{\mu_{12}^2}{\sin\beta\cos\beta}[1 + 3\cot^2\beta]. \quad (13)$$

Let Φ_1 couple to the d, s, b quarks and e, μ, τ leptons, and Φ_2 to the u, c, t quarks. Then the main contribution to C_1 from fermions is $12 m_b^2/\cos^2\beta$, and that to C_2 is $12 m_t^2/\sin^2\beta$. Hence $C_1 < 0$ and $C_2 > 0$ over most of the interesting parameter space. Consider for example $m_{H^\pm} = 150$ GeV and $m_A = 100$ GeV. Using

$$\left[\begin{array}{c} m_1^2 \\ m_2^2 \end{array}\right] = \frac{1}{A_{11}A_{22} - A_{21}A_{12}} \left[\begin{array}{cc} A_{22} & -A_{12} \\ -A_{21} & A_{11} \end{array}\right] \left[\begin{array}{c} C_1 \\ C_2 \end{array}\right], \tag{14}$$

$m_{1,2}$ may then be obtained as functions of μ_{12}^2, $\tan\beta$, and $\tan\alpha$. There are 2 special limits for μ_{12}^2, namely zero and $\mu_{12}^2 = -m_A^2 \sin\beta\cos\beta$ for which $\lambda_5 = 0$. In the following numerical analysis, only these 2 limits will be considered. Although the parameters $\tan\alpha$ and $\tan\beta$ are in principle independent, it turns out that solutions only exist if they are small and not too far apart in value. Hence they will also be set equal here for simplicity. In Table 1, m_1 and m_2 are shown for various values of $\tan\beta = \tan\alpha$ for the cases $\mu_{12}^2 = 0$ and $\lambda_5 = 0$.

	$\mu_{12}^2 = 0$	$\lambda_5 = 0$
$\tan\beta$	(m_1, m_2) [GeV]	(m_1, m_2) [GeV]
0.2	(154, 355)	(172, 369)
0.3	(150, 365)	(168, 378)
0.4	(142, 380)	(162, 393)
0.5	(129, 402)	(151, 414)
0.6	(106, 435)	(131, 446)
0.7	(39, 487)	(87, 497)

Table 1. Values of $m_{1,2}$ from $\tan\beta = \tan\alpha$ for $\mu_{12}^2 = 0$ and $\lambda_5 = 0$.

It has thus been demonstrated that in a class of two-Higgs-doublet models defined by Eq. (2), the cancellation of quadratic divergences is possible [i.e. Eqs. (3) and (4)] with realistic values of the Higgs-boson masses, even though the right-hand-side of Eq. (3) is negligible because of small Yukawa couplings. [Whereas $\lambda_1 > 0$ is required, $2\lambda_3 + \lambda_4 < 0$ is allowed.] In particular, the numerical values of Table 1 show that m_1 of order 100 GeV [5] is not impossible.

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References

[1] M. Veltman, Acta Phys. Pol. **B12**, 437 (1981).

[2] C. Newton and T. T. Wu, Z. Phys. **C62**, 253 (1994).

[3] See for example E. Ma, in *Particle Phenomenology in the 90's*, edited by A. Datta, P. Ghose, and A. Raychaudhuri (World Scientific, Singapore, 1992), p. 383.

[4] See for example T. V. Duong *et al.*, Phys. Rev. **D52**, 5045 (1995).

[5] ALEPH Collaboration, Phys. Lett. **B495**, 1 (2000); L3 Collaboration, Phys. Lett. **B495**, 18 (2000).